

# Variable Chaplygin Gas : Constraints from Look-Back Time and SNe Ia(Union 2.1 Compilation)

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## Abstract

In this paper we try to constrain the parameters of the Variable Chaplygin Gas Model, with equation of state  $P = -A(a)/\rho$ , where  $A(a)$  is a positive function of the scale factor from lookback time measurements of high- $z$  galaxies catalogued in the Gemini Deep Deep Survey(GDDS) and SNe Ia Union2.1[1] Compilation provided by the Supernova Cosmology Project(SCP).

## 1 Introduction

Observations including the Hubble diagram of Type Ia Supernovae(SNe Ia)[2], Cosmic Microwave Background and galaxy spectra establish that the expansion of the universe is accelerating and that two-thirds of the total energy density is in the form of dark energy with negative pressure. The most viable candidate of dark energy is the cosmological constant  $\Lambda$  with a time constant equation of state.

However this model suffers from serious fine tuning problems[3, 4]. Hence many more models have been explored in the past and this remains an active area of research. Recently, an alternative class of models have been proposed which involve a slowly evolving and spatially homogenous scalar field[6, 7] or two coupled fields[8]. However, these “Quintessence” models also suffer from fine tuning problem.

As an alternative to both the cosmological constant and quintessence, it is also possible to explain the accelerating expansion of the universe by introducing a cosmic fluid component with an exotic equation of state, called Chaplygin gas[9, 10]. The attractive feature of such models is that they can explain both

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dark energy and dark matter with a single component. The equation of state for the Chaplygin gas is  $P = -A/\rho$ , where  $A$  is a positive constant. A more generalized model of Chaplygin gas is characterised by an equation of state

$$P_{ch} = -\frac{A}{(\rho_{ch})^\alpha} \quad (1)$$

where  $\alpha$  is a constant in the range  $0 < \alpha \leq 1$  (the Chaplygin gas corresponds to  $\alpha = 1$ ). The energy conservation law requires that the energy density of the Chaplygin gas evolves as [11]

$$\rho_{ch} = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \quad (2)$$

where  $a$  is a scale factor and  $B$  is the constant of integration. The above equation indicates that the Chaplygin gas behaves like non relativistic matter at early times while at later times the equation of state is dominated by a cosmological constant  $8\pi G A^{1/(1+\alpha)}$  which leads to the observed accelerated expansion. Models based on Chaplygin gas have been found to be consistent with SNe Ia data [12], CMB peak locations [13] and other observational tests like gravitational lensing, cosmic age of old high red shift objects etc. [14], as also with some combination of some of them [15]. It has been shown that this model can be accommodated within the standard structure formation scenarios [10, 11, 16]. Therefore the Chaplygin gas model seems to be a good alternative to explain the accelerated expansion of the universe. The main problem of the Chaplygin gas model was that it produced oscillations or exponential blowup of matter power spectrums that were inconsistent with observations [17]. However this instability can be overcome by considering the joint effect of shear and rotation that slows down the collapse with respect to the simple spherical collapse model [18].

Subsequently a *variable* Chaplygin gas model was proposed [19] and constrained using SNe Ia “gold” data [20]. In this paper we try to constrain the variable Chaplygin gas model parameters using lookback time analysis and latest SNe Ia Union2.1 data set. The basic formalism of the model is reviewed in Section 2. Section 3 gives the general theory for lookback time. Analysis of Supernovae Type Ia data is presented in Section 4. We discuss the results and conclusions in Section 5.

## 2 Variable Chaplygin Gas Model

The variable Chaplygin gas [19] is characterised by the equation of state :

$$P_{ch} = -\frac{A(a)}{\rho_{ch}} \quad (3)$$

where  $A(a) = A_0 a^{-n}$  is a positive function of the cosmological scale factor  $a$ .  $A_0$  and  $n$  are constants. Using the energy conservation equation in a flat

Friedman-Robertson-Walker universe and equation (3), the variable Chaplygin gas density evolves as :

$$\rho_{ch} = \sqrt{\frac{6}{6-n} \frac{A_0}{a^n} + \frac{B}{a^6}} \quad (4)$$

where B is a constant of integration.

The original Chaplygin gas behavior is restored for  $n = 0$  and the gas behaves initially as dust-like matter and later as a cosmological constant. However, in the present case the Chaplygin gas evolves from dust dominated epoch to quintessence in present times (see [19])

The Friedman equation for a spatially flat universe is:

$$H^2 = \frac{8\pi G}{3} \rho \quad (5)$$

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter. Therefore the acceleration condition  $\ddot{a} > 0$  is equivalent to

$$\left(3 - \frac{6}{6-n}\right) a^{6-n} > \frac{B}{A_0} \quad (6)$$

which requires  $n < 4$ . This gives the present value of energy density of the variable Chaplygin gas

$$\rho_{cho} = \sqrt{\frac{6}{6-n} A_0 + B} \quad (7)$$

where  $a_0 = 1$ . Defining

$$\Omega_m = \frac{B}{6A_0/(6-n) + B} \quad (8)$$

the energy density becomes

$$\rho_{ch}(a) = \rho_{ch0} \left[ \frac{\Omega_m}{a^6} + \frac{1 - \Omega_m}{a^n} \right]^{1/2} \quad (9)$$

## 3 Lookback Time

### 3.1 Theory

We consider a homogenous, isotropic, spatially flat universe described by the Friedman-Robertson-Walker metric with the line element  $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ , where  $a(t)$  is the cosmological scale factor. The speed of light has been set to unity in our analysis.

The Lookback time is defined as the difference between the age of the Universe today ( $t_0$ ) and its age ( $t_z$ ) at redshift  $z$ . The relation between the Lookback

time and redshift  $z$  can be written as

$$t_L(z; \mathbf{p}) = H_0^{-1} \int_0^z \frac{dz'}{(1+z')\mathcal{H}(\mathbf{p})} \quad (10)$$

where  $H_0$  is the Hubble constant,  $\mathbf{p}$  denotes the set of all parameters in the cosmological model and  $\mathcal{H}(\mathbf{p})$  denotes the Hubble parameter as defined in the model chosen.

The observed lookback time to an object, a galaxy cluster for example, at redshift  $z_i$  is defined as (see [21])

$$t_{L,obs}(z_i; \tau) = t_{obs}^0 - t(z_i) - \tau \quad (11)$$

where

$t_{obs}^0$  is the measured total expanding age of the Universe,

$t(z_i)$  is the age of the object-defined as the difference between the present age of the Universe and the age of the Universe when the object was born at redshift  $z_f$ .

$\tau$  denotes the incubation time or delay factor which accounts for our lack of knowledge about the amount of time since the beginning of the structure formation in the Universe until the formation time ( $t_f$ ) of the object.

### 3.2 Data, Statistical Analysis and Results

In order to constrain the parameters of the variable Chaplygin model we do a statistical analysis of the Gemini Deep Deep Survey (GDDS) lookback time estimates of high  $z$ -galaxies. The total GDDS sample consists of 20 old passive galaxies distributed over the redshift range  $1.308 \leq z \leq 2.147$  (McCarthy et al, 2004 [22])

However we build our LT sample using the criterion discussed by Dantas et al [23]-given two objects at approximately the same  $z$ , the older one is selected. Going by this criterion, a set of 8 data points is obtained. These are tabulated in Table 1. Another important aspect of the data set, as pointed out by Dantas et al is that the total age of the Universe is assumed to be  $t_0^{obs} = 13.6_{-0.3}^{+0.4} Gyr$ . This is the result obtained by MacTavish et al. (2005) from a joint analysis involving the most recent CMB experiments.

To estimate the best fit to the parameter set  $\mathbf{p} \equiv \{\Omega_m, n\}$  the likelihood function is defined

$$L_{age} \propto \exp[-\chi_{age}^2(z; \mathbf{p}, \tau)/2], \quad (12)$$

where

$$\chi_{age}^2 = \sum_i \frac{[t_L(z_i; \mathbf{p}) - t_L^{obs}(z_i; \tau)]^2}{\sigma_i^2 + \sigma_{t_0^{obs}}^2} + \frac{[t_0(\mathbf{p}) - t_0^{obs}]^2}{\sigma_{t_0^{obs}}^2} \quad (13)$$

Here,  $\sigma_i^2$  is the uncertainty in the individual lookback time to the  $i^{th}$  galaxy of our sample and  $\sigma_{t_0^{obs}}^2 = 0.35$  Gyr is the uncertainty on the total expanding age of the Universe( $t_0^{obs}$ )

Since galaxies form at different epochs, in principle, the value for the incubation time  $\tau$  for each object in the sample is expected to vary. However since for the given data we do not know the formation redshift( $z_f$ ) for each object, we treat the incubation time as a “nuisance” parameter and marginalise over it.

To this effect a modified log-likelihood function may be defined and an analytical marginalisation may be carried out-

$$\begin{aligned}\tilde{\chi}_{age}^2 &= -2\ln \int_0^\infty d\tau \exp\left(-\frac{1}{2}\chi_{age}^2\right) \\ &= A - \frac{B^2}{C} + D - 2\ln \left[ \sqrt{\frac{\pi}{2C}} \operatorname{erfc}\left(\frac{B}{\sqrt{2C}}\right) \right],\end{aligned}\tag{14}$$

where

$$A = \sum_i \frac{\Delta^2}{\sigma_T^2}, \quad B = \sum_i \frac{\Delta}{\sigma_T^2} \quad C = \sum_i \frac{1}{\sigma_T^2},$$

$D$  is the second term of the RHS of Eqn. 13  $\Delta = t_L(z_i; p) - [t_0^{obs} - t(z_i)]$

and  $\operatorname{erfc}(x)$  is the complementary error function over the variable  $x$ .

Figures 1, 2 and 3 show the results of our analysis.  $1\sigma$  (68.3%),  $2\sigma$  (95.4%) and  $3\sigma$  (99.7%) confidence regions are shown. We see that the  $3\sigma$  contour plots constrain the parameter space  $\Omega_m - n$  to  $\Omega_m \in [0.0, 0.187]$  and  $n \in [-4, 4]$ . The best fit parameters are obtained at  $\Omega_m = 0.0325$  and  $n = 2.3467$  with the corresponding  $\chi_{min}^2 = 7.4759$ .

## 4 SNe Ia Data and $\chi^2$ Minimization

SNe Ia play a key role in our understanding of the accelerated expansion of the Universe. We consider the constraints on parameters of the variable Chaplygin gas model through a statistical analysis involving the most recent SNe Ia data, as provided by the Supernova Cosmology Project (SCP) Union 2.1 Compilation, consisting of 580 SNe from various surveys providing redshift, distance moduli and associated errors in distance moduli. The theoretical distance moduli of supernovae is dependent on the cosmological model and can be used to constrain the cosmological parameters of the model in consideration.

To find out the luminosity distance the contributions from radiation and baryons together with the Chaplygin gas have been taken into account.

Using the Friedman Equation, The luminosity distance in a flat universe can be expressed as:

$$d_L = \frac{c}{aH_0} \int_a^1 \frac{da}{\Omega_{cho}^{1/2} X(a)}\tag{15}$$

and the distance modulus

$$\mu_{th} = 5 \log \frac{H_0 d_L}{ch} + 42.38 \quad (16)$$

The best fit parameters are determined by minimising

$$\chi^2 = \sum_{i=1}^{580} \left[ \frac{\mu_{th}^i - \mu_{obs}^i}{\sigma_i} \right]^2 - \frac{\sum_{i=1}^{580} \left[ \frac{\mu_{th}^i - \mu_{obs}^i}{\sigma_i^2} \right]}{\sum_{i=1}^{580} \left[ \frac{1}{\sigma_i^2} \right]} \left( \sum_{i=1}^{580} \left[ \frac{\mu_{th}^i - \mu_{obs}^i}{\sigma_i^2} \right] + \frac{2}{5} \ln 10 \right) - 2 \ln h \quad (17)$$

We obtain the best fit parameters at  $\Omega_m = 0.146$  and  $n = -1.2$  with the corresponding  $\chi_{min}^2 = 580.12$  for 579 degrees of freedom.

Figures 4,5 and 6 show the results of our SNe Ia data analysis.  $1\sigma$  (68.3%),  $2\sigma$  (95.4%) and  $3\sigma$  (99.7%) confidence regions are shown. We note that the  $3\sigma$  contour plots constrain the parameter space  $\Omega_m - n$  to  $\Omega_m \in [0.09, 0.226]$  and  $n \in [-3.7, 0.35]$ .

## 5 Discussion

We see that the variable Chaplygin gas model is able to account for the observations regarding the evolution of the universe. Initially the gas behaves like non-relativistic matter and later accounts for the accelerated expansion of the universe, which describes the current state of the universe. Guo and Zhang[19] tested the consistency of the variable Chaplygin model for the SNe Ia “gold” data set[20] and obtained the best fit parameters for  $\Omega_m = 0.25$  and  $n = -2.9$  with the corresponding  $\chi_{min}^2 = 173.88$ . This test was repeated by Sethi et. al.[25], but they also took into account contribution from matter and radiation. They obtained similar results with  $\Omega_m = 0.22$  and  $n = -2.8$  at  $\chi_{min}^2 = 174.36$ . For  $3\sigma$  contours they determined the parameter space to be restricted to  $\Omega_m \in [0.0, 0.36]$  and  $n \in [-41.3, 2.8]$ .

With our analysis of the Union2.1 data and look back time and measurement of the age of the Universe, we see that the ranges obtained for  $\Omega_m$  and  $n$  not only lie within the confidence levels obtained by Sethi et. al. [25] but we have further constrained the possible values of  $\Omega_m$  and  $n$  by a joint analysis of the SNe Ia Union2.1 and look-back time data.

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$z$	$Age(Gyr)$
1.396	$4.0^{+0.4}_{-3.5}$
1.450	$3.2^{+0.7}_{-2.8}$
1.490	$3.0^{+0.7}_{-0.2}$
1.493	$3.4^{+0.3}_{-1.7}$
1.646	$2.6^{+0.3}_{-0.3}$
1.725	$2.1^{+0.4}_{-0.9}$
1.801	$0.9^{+0.5}_{-0.2}$
2.147	$1.2^{+0.1}_{-0.4}$

Table 1: Redshift-Age data points[22]

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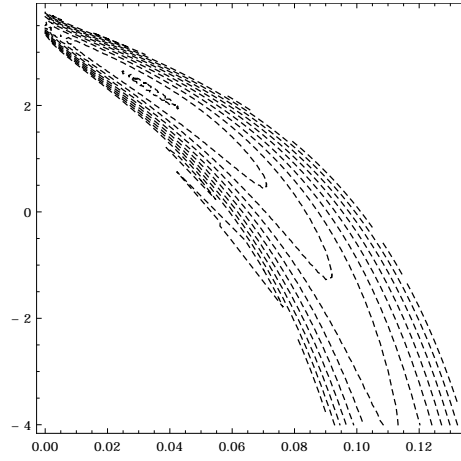


Figure 1:  $1\sigma$   $\chi^2$  contours for GDDS look-back time data and measurement of the age of the Universe time on  $\Omega_m - n$  parameter space

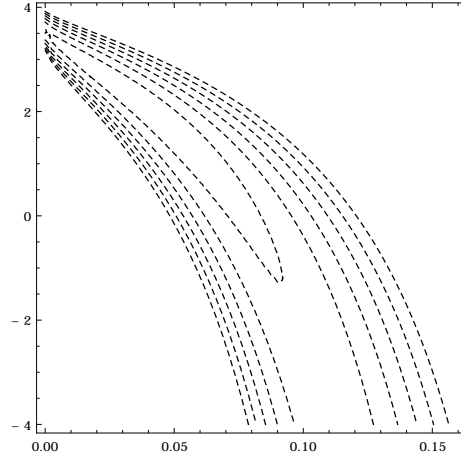


Figure 2:  $2\sigma$   $\chi^2$  contours for GDDS look-back time data and measurement of the age of the Universe time on  $\Omega_m - n$  parameter space

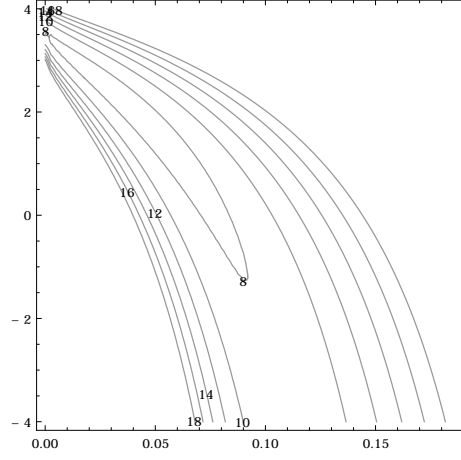


Figure 3:  $3\sigma \chi^2$  contours for GDDS look-back time data and measurement of the age of the Universe time on  $\Omega_m - n$  parameter space

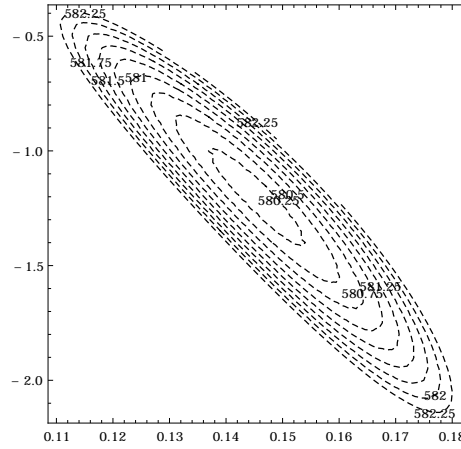


Figure 4:  $1\sigma \chi^2$  contours for SNe Ia Union2.1 data on  $\Omega_m - n$  parameter space

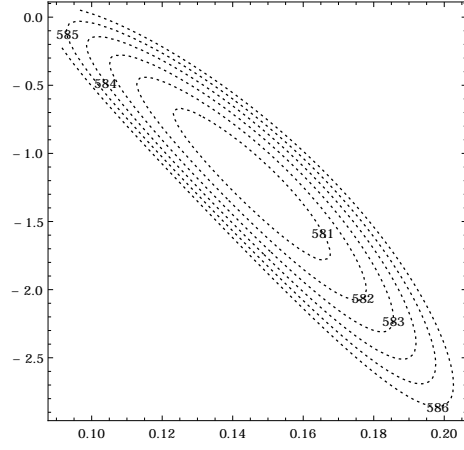


Figure 5:  $2\sigma \chi^2$  contours for *SNe Ia Union2.1* on  $\Omega_m - n$  parameter space

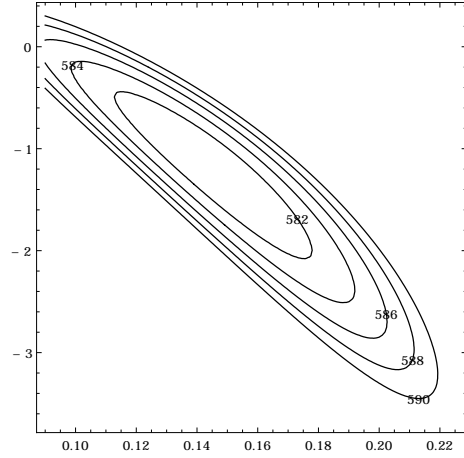


Figure 6:  $3\sigma \chi^2$  contours for *SNe Ia Union2.1* on  $\Omega_m - n$  parameter space